4595. *Proposed by Nguyen Viet Hung.*

Let $n > 2$ be an integer and let $S_n = \sum_{k=2}^{n} \sqrt{1+\frac{2}{k^2}}$. Determine $\lfloor S_n \rfloor$.

There were 21 solutions submitted, all correct. We present the solution by Arkady Alt and Florentin Visescu (done independently).

We have:

$$\begin{aligned} n-1 &< \sum_{k=2}^{n} \sqrt{1+\frac{2}{k^2}} < \sum_{k=2}^{n}\left(1+\frac{1}{k^2}\right) \\ &< \sum_{k=2}^{n}\left(1+\frac{1}{(k-1)k}\right) = (n-1) + \sum_{k=2}^{n}\left(\frac{1}{k-1}-\frac{1}{k}\right) \\ &= (n-1)+1-\frac{1}{n} < n, \end{aligned}$$

from which it follows that $\lfloor S_n \rfloor = n-1$.

Comment from the editor. Seven solvers used the upper bound $(\pi^2/6)-1$ and two the upper bound $\int_1^\infty x^{-2}\,dx = 1$ for $\sum_{k=2}^{n} k^{-2}$. Two solvers used the interesting identity

$$\sqrt{1+\frac{1}{(x-1)^2}+\frac{1}{x^2}} = 1+\frac{1}{x-1}-\frac{1}{x}.$$

4596. *Proposed by Boris Čolaković.*

Let a, b, c be the lengths of the sides of triangle ABC with inradius r and circumradius R. Show that

$$\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b} \leq \frac{R}{r}-\frac{1}{2}$$

We received 39 solutions, all of which were correct. Of these solutions, 20 were submitted by Vivek Mehra. We present the solution by Nguyen Viet Hung.

The desired inequality is succesively equivalent to

$$\frac{a(a+b)(a+c)+b(b+c)(b+a)+c(c+a)(c+b)}{(a+b)(b+c)(c+a)}+\frac{1}{2} \leq \frac{R}{r},$$

$$\frac{a^3+b^3+c^3+(a+b+c)(ab+bc+ca)}{(a+b+c)(ab+bc+ca)-abc}+\frac{1}{2} \leq \frac{R}{r},$$

$$\frac{3abc+(a+b+c)(a^2+b^2+c^2-ab-bc-ca)+(a+b+c)(ab+bc+ca)}{(a+b+c)(ab+bc+ca)-abc}+\frac{1}{2} \leq \frac{R}{r},$$

$$\frac{3abc+(a+b+c)(a^2+b^2+c^2)}{(a+b+c)(ab+bc+ca)-abc}+\frac{1}{2} \leq \frac{R}{r}.$$